

Contents

Preface	i
1 What is Logic?	1
1.1 Logical consequence and logical truth	2
1.2 Formalization	3
1.3 Metalogic	4
<i>Exercises 1.1–1.2</i>	8
1.4 Application	8
1.5 The nature of logical consequence	9
<i>Exercise 1.3</i>	11
1.6 Logical constants	12
1.7 Extensions, deviations, variations	14
1.8 Set theory	15
<i>Exercises 1.4–1.5</i>	29
2 Propositional Logic	30
2.1 Grammar of PL	30
2.2 The semantic approach to logic	34
2.3 Semantics of propositional logic	35
<i>Exercise 2.1</i>	42
2.4 Validity and invalidity in PL	43
<i>Exercise 2.2</i>	44
2.4.1 Schemas, validity, and invalidity	45
2.5 Sequent proofs in PL	46
2.5.1 Sequents	47
2.5.2 Rules	50
2.5.3 Sequent proofs	52
2.5.4 Example sequent proofs	54
<i>Exercise 2.3</i>	57

2.6	Axiomatic proofs in PL	57
	<i>Exercise 2.4</i>	62
2.7	Soundness of PL and proof by induction	62
	<i>Exercises 2.5–2.10</i>	70
2.8	PL proofs and the deduction theorem	71
	<i>Exercises 2.11–2.12</i>	78
2.9	Completeness of PL	78
2.9.1	Maximal consistent sets of wffs	79
2.9.2	Maximal consistent extensions	80
2.9.3	Features of maximal consistent sets	82
2.9.4	The proof	83
3	Beyond Standard Propositional Logic	85
3.1	Alternate connectives	85
3.1.1	Symbolizing truth functions in propositional logic	85
3.1.2	Sheffer stroke	88
3.1.3	Inadequate connective sets	89
	<i>Exercises 3.1–3.3</i>	90
3.2	Polish notation	90
	<i>Exercise 3.4</i>	91
3.3	Nonclassical propositional logics	91
3.4	Three-valued logic	93
3.4.1	Łukasiewicz's system	95
	<i>Exercises 3.5–3.6</i>	98
3.4.2	Kleene's tables	98
	<i>Exercises 3.7–3.9</i>	100
3.4.3	Determinacy	100
3.4.4	Priest's logic of paradox	102
	<i>Exercises 3.10–3.11</i>	104
3.4.5	Supervaluationism	104
	<i>Exercises 3.12–3.16</i>	110
3.5	Intuitionistic propositional logic: proof theory	110
	<i>Exercise 3.17</i>	114
4	Predicate Logic	115
4.1	Grammar of predicate logic	115
4.2	Semantics of predicate logic	117
	<i>Exercise 4.1</i>	122

4.3	Establishing validity and invalidity	123
	<i>Exercises 4.2–4.3</i>	126
4.4	Axiomatic proofs in PC	126
	<i>Exercise 4.4</i>	133
4.5	Metalogic of PC	134
	<i>Exercise 4.5</i>	136
5	Beyond Standard Predicate Logic	137
5.1	Identity	137
5.1.1	Grammar for the identity sign	138
5.1.2	Semantics for the identity sign	138
5.1.3	Symbolizations with the identity sign	139
	<i>Exercises 5.1–5.2</i>	140
5.2	Function symbols	141
	<i>Exercise 5.3</i>	143
5.2.1	Grammar for function symbols	143
5.2.2	Semantics for function symbols	144
	<i>Exercise 5.4</i>	145
5.3	Definite descriptions	146
5.3.1	Grammar for ι	146
5.3.2	Semantics for ι	147
	<i>Exercises 5.5–5.6</i>	150
5.3.3	Elimination of function symbols and descriptions	150
	<i>Exercises 5.7–5.8</i>	153
5.4	Further quantifiers	153
5.4.1	Generalized monadic quantifiers	154
	<i>Exercise 5.9</i>	156
5.4.2	Generalized binary quantifiers	156
	<i>Exercise 5.10</i>	158
5.4.3	Second-order logic	158
	<i>Exercise 5.11</i>	162
5.5	Complex Predicates	162
	<i>Exercises 5.12–5.13</i>	165
5.6	Free Logic	165
5.6.1	Semantics for free logic	166
	<i>Exercises 5.14–5.15</i>	169
5.6.2	Proof theory for free logic	170

6	Propositional Modal Logic	171
6.1	Grammar of MPL	174
6.2	Symbolizations in MPL	174
6.3	Semantics for MPL	176
6.3.1	Kripke models	178
	<i>Exercise 6.1</i>	184
6.3.2	Semantic validity proofs	184
	<i>Exercise 6.2</i>	187
6.3.3	Countermodels	187
	<i>Exercise 6.3</i>	203
6.4	Axiomatic systems of MPL	203
6.4.1	System K	204
	<i>Exercises 6.4–6.5</i>	213
6.4.2	System D	213
	<i>Exercise 6.6</i>	214
6.4.3	System T	214
	<i>Exercise 6.7</i>	215
6.4.4	System B	215
	<i>Exercise 6.8</i>	216
6.4.5	System S ₄	216
	<i>Exercise 6.9</i>	218
6.4.6	System S ₅	218
	<i>Exercise 6.10</i>	219
6.4.7	Substitution of equivalents and modal reduction	219
	<i>Exercise 6.11</i>	221
6.5	Soundness in MPL	222
	<i>Exercises 6.12–6.13</i>	224
6.5.1	Soundness of K	224
6.5.2	Soundness of T	224
6.5.3	Soundness of B	224
	<i>Exercises 6.14–6.15</i>	225
6.6	Completeness in MPL	225
6.6.1	Definition of canonical models	226
6.6.2	Facts about maximal consistent sets	227
	<i>Exercise 6.16</i>	229
6.6.3	“Mesh”	229
	<i>Exercise 6.17</i>	231
6.6.4	Truth and membership in canonical models	231

6.6.5	Completeness of systems of MPL	232
	<i>Exercises 6.18–6.20</i>	233
7	Beyond Standard MPL	234
7.1	Deontic logic	234
	<i>Exercises 7.1–7.2</i>	237
7.2	Epistemic logic	237
	<i>Exercise 7.3</i>	238
7.3	Propositional tense logic	239
7.3.1	The metaphysics of time	239
7.3.2	Tense operators	241
7.3.3	Kripke-style semantics for tense logic	242
	<i>Exercises 7.4–7.5</i>	243
7.3.4	Formal constraints on \leq	243
	<i>Exercise 7.6</i>	246
7.4	Intuitionistic propositional logic: semantics	246
7.4.1	Proof stages	246
	<i>Exercises 7.7–7.8</i>	249
7.4.2	Examples	249
	<i>Exercises 7.9–7.10</i>	251
7.4.3	Soundness	251
	<i>Exercises 7.11–7.13</i>	253
8	Counterfactuals	254
8.1	Natural language counterfactuals	255
8.1.1	Antecedents and consequents	255
8.1.2	Can be contingent	255
8.1.3	No augmentation	256
8.1.4	No contraposition	256
8.1.5	Some implications	257
8.1.6	Context dependence	257
8.2	The Lewis/Stalnaker theory	260
8.3	Stalnaker’s system (SC)	261
8.3.1	Syntax of SC	261
8.3.2	Semantics of SC	261
	<i>Exercise 8.1</i>	264
8.4	Validity proofs in SC	264
	<i>Exercise 8.2</i>	265

8.5	Countermodels in SC	266
	<i>Exercises 8.3–8.4</i>	275
8.6	Logical Features of SC	275
8.6.1	No exportation	276
8.6.2	No importation	277
8.6.3	No transitivity	278
8.6.4	No transposition	278
8.7	Lewis’s criticisms of Stalnaker’s theory	279
8.8	Lewis’s system	282
	<i>Exercises 8.5–8.6</i>	284
8.9	The problem of disjunctive antecedents	284
9	Quantified Modal Logic	286
9.1	Grammar of QML	286
9.2	De re and de dicto	286
9.3	A simple semantics for QML	290
9.4	Countermodels and validity proofs in SQML	292
	<i>Exercise 9.1</i>	297
9.5	Philosophical questions about SQML	298
9.5.1	The necessity of identity	298
9.5.2	The necessity of existence	300
	<i>Exercise 9.2</i>	304
9.5.3	Necessary existence defended	304
9.6	Variable domains	307
9.6.1	Contingent existence vindicated	310
	<i>Exercises 9.3–9.4</i>	310
9.6.2	Increasing, decreasing domains	310
	<i>Exercise 9.5</i>	311
9.6.3	Strong and weak necessity	312
9.6.4	Actualist and possibilist quantification	314
9.7	Axioms for SQML	315
	<i>Exercise 9.6</i>	318
10	Two-dimensional modal logic	319
10.1	Actuality	319
10.1.1	Kripke models with designated worlds	320
	<i>Exercise 10.1</i>	321
10.1.2	Semantics for @	321

10.1.3	Establishing validity and invalidity	321
10.2	\times	322
10.2.1	Two-dimensional semantics for \times	323
	<i>Exercise 10.2</i>	326
10.3	Fixedly	326
	<i>Exercises 10.3–10.5</i>	327
10.4	Necessity and <i>a priori</i>	328
	<i>Exercises 10.6–10.9</i>	334
A	Answers and Hints	335
	References	353
	Index	359