Consequences of collapse*

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Forthcoming in Donald Baxter and Aaron Cotnoir, eds., *Composition as Identity*

Composition as identity is the strange and strangely compelling doctrine that the whole is in some sense identical to its parts. According to the most interesting and fun version, the one inspired[1] by Donald Baxter, this is meant in the most straightforward way: a single whole is genuinely identical to its many parts taken together—identical in the very same sense of 'identical', familiar to philosophers, logicians, and mathematicians, in which I am identical to myself and $2+2$ is identical to 4.

Composition as identity implies the principle of Collapse: something is one of the Xs iff it is part of the fusion of the Xs. (Collapse is so-called because it in effect identifies mereologically equivalent pluralities.) In an earlier paper I pointed out that Collapse alters Boolos's logic of plural quantification in various ways.[2] Here I point out some further consequences of Collapse. For example, collapse implies that plural definite descriptions do not function normally. (As we will see, this undermines Kris McDaniel's (2008) recent argument against composition as identity.) Also it opens the door to drastic—albeit unattractive—ideological simplifications: parthood, identity, and the plural quantifiers may all be eliminated.

1. Composition as identity formulated

Composition as identity is a logically radical thesis, since it holds that a single thing can be identical to many things (its parts). Its formulation, therefore, calls for some nonstandard logical ideology.

Begin with the primitive notions of first order logic, plus plural quantifiers and variables (symbolize "for some Xs" as $\exists X$), plus 'y is one of the Xs'

*Part of this is a revised version of my unpublished paper "Composition as Identity and Emergent Properties: Reply to McDaniel". Thanks to Don Baxter, Dan Giberman, Kris McDaniel, and David Nicolas for feedback.

[1]Baxter (1988*a*,*b*). I say "inspired" because Baxter's own view is even more radical; see Baxter (2012).

[2]Sider (2007). On that basis I rejected (fun, interesting) composition as identity, but I no longer find that argument convincing since I now doubt that Boolos's logic should be taken as metaphysically basic (Sider, 2011, section 9.12.4).

(symbolized: Xy), plus a predicate for parthood, $<$, plus an identity predicate $=$ where the flanking variables may each be either plural or singular. Define overlap and fusion as usual:

$$Oxy =_{\text{df}} \exists z(z<x \wedge z<y)$$
$$x \text{ Fu } Y =_{\text{df}} \forall z(Yz \rightarrow z<x) \wedge \forall z(z<x \rightarrow \exists w(Yw \wedge Ozw))$$

(Objects overlap when they share a part in common; x is a fusion of the Ys iff anything that is one of the Ys is part of x, and each part of x overlaps something that is one of the Ys.) Composition as identity may then be formulated as follows:

$$\forall x \forall Y(x \text{ Fu } Y \rightarrow x = Y) \qquad \text{(Composition as identity)}$$

Composition as identity, as I will understand it, includes some further assumptions. For reasons that will become clear below, in order to state those assumptions I need to introduce a second notion of fusion. Let ϕ be any formula, v any variable, and $\phi_v(v')$ the result of changing free vs to free v's in ϕ. The notion of an S-fusion (S for schematic) may then be defined as follows:

$$x \text{ S-Fu}_v\ \phi =_{\text{df}} \forall z(\phi_v(z) \rightarrow z<x) \wedge \forall z(z<x \rightarrow \exists w(\phi_v(w) \wedge Ozw))$$

(x is an S-fusion of the ϕs iff each ϕ is part of x and each part of x overlaps some ϕ). S-fusion is "schematic" because the variables ϕ and v in the definition are not in the object language, but are rather metalinguistic; the definition supplies a definiens whenever ϕ and v are replaced with a formula and variable of the object language.

Now for the assumptions. First, classical first-order mereology, including both of the following fusion principles in addition to reflexivity, transitivity, antisymmetry, and strong supplementation:[3]

$$\exists y \phi_v(y) \rightarrow \exists y\ y \text{ S-Fu}_v\ \phi \qquad \text{(fusions, schematic form)}$$
$$\forall Y \exists x\ x \text{ Fu } Y \qquad \text{(fusions, plural form)}$$

Second:

$$\alpha = \beta, \psi(\alpha) \vdash \psi(\beta) \qquad \text{(Leibniz's Law)}$$

[3]The schematic form implies the plural, letting ϕ be Xv. (Indeed, we could define "x Fu Y" as meaning "x S-Fu$_v$ Yv".) I argued in Sider (2007, section 3.2) that composition as identity implies all of classical mereology except for unrestricted composition, but my arguments employed a plural logic with a primitive plural-term-forming operator 'and'. I don't see how to make analogous arguments in the present context.

And third:

$$\exists y \phi_v(y) \rightarrow \exists X \exists z(z \text{ S-Fu}_v\ \phi \wedge \forall y(Xy \leftrightarrow y{<}z)) \qquad \text{(Comprehension)}$$

Leibniz's Law here is intended in a very strong form: it is intended to apply even when one of α and β is singular and the other is plural. This requires expanding the language's grammar to make the substitutions grammatical—predicates cannot be fixedly singular or plural, for example. If composition as identity is to be taken at face value, as holding that a thing is genuinely identical to its many parts, then Leibniz's Law must be taken in this strong form.[4]

2. Collapse

Composition as identity, together with the assumptions listed in the previous section, implies Collapse:[5]

$$\forall X \forall z(z \text{ Fu } X \rightarrow \forall y(Xy \leftrightarrow y{<}z)) \qquad \text{(Collapse)}$$

("y is one of the Xs iff y is part of the fusion of the Xs"). For suppose z Fu X, and take any y. Suppose first that Xy; then by the definition of 'Fu', $y{<}z$. Conversely, suppose $y{<}z$. By comprehension, for some Ys and some o, (i) o S-Fu$_v$ $v{=}y \vee v{=}z$ and (ii) $\forall w(Yw \leftrightarrow w{<}o)$. By tedious mereology, $o = z$. (By (i), $z < o$. Also, by (i), every part of o overlaps either z or y, and so overlaps z; so $o < z$ by strong supplementation. So by antisymmetry, $o = z$.) Then, by further tedious mereology, z Fu Y. (Take any w such that Yw; then by (ii), $w < o$; so, $w < z$. Next, take any w such that $w < z$; then $w < o$; so by (ii), Yw; so w overlaps one of the Ys—namely, itself.) Now, since z Fu X and z Fu Y, by composition as identity, $z = X$ and $z = Y$; and hence, by Leibniz's Law, $X = Y$. But by (i), $y < o$; so by (ii), Yy; so by Leibniz's Law, Xy.

Given Collapse, there are fewer pluralities than one normally expects. There are, for example, no Xs such that something is one of them if and only if it is a human being. For any Xs including all humans will also include some nonhumans, and thus will not include only humans. If each human is one of the Xs then the fusion of the Xs (which must exist given the plural fusions principle) contains many non-human parts (nonhuman parts of individual humans, and nonhuman objects containing multiple humans as parts, for example), and each

[4] See Sider (2007, section 3.1).

[5] See Sider (2007, section 3.2).

non-human part of the fusion of the Xs must be one of the Xs given Collapse. In general, pluralities that include *all* Fs will not include *only* Fs, except in the special case where each part of the fusion of all Fs (i.e., each part of the x such that $x\,\text{S-Fu}_v F v$) is itself an F.

3. Comprehension

First consequence of Collapse: the comprehension principle for plural logic needs to be weakened. Boolos's plural language is usually supposed to have a logic analogous to monadic impredicative second-order logic (minus the empty plurality), and thus to obey the following principle:

$$\exists y \phi \rightarrow \exists X \forall y (Xy \leftrightarrow \phi) \qquad \text{(Comprehension, usual version)}$$

("provided there's at least one ϕ, there are some Xs such that something is one of them iff it is a ϕ"). But this stronger form of comprehension fails given Collapse. As we just saw, Collapse prohibits there being Xs such that something is one of them iff it is a human being. The comprehension principle stated in section 1 was weaker; it says merely that, provided there's at least one ϕ, there are some Xs such that something is one of them iff it is a part of the S-fusion of the ϕs.

The reason for introducing the second notion of fusion, S-fusion, can now be given: it is needed to state the weakened comprehension principle. We could not have stated that principle using the nonschematic, plural notion of fusion: "provided there's at least one ϕ, there are some Xs such that something is one of them iff it is a part of the [nonschematic] fusion of the ϕs". For what are "the ϕs"? Some Ys such that something is one of them iff it is a ϕ, presumably; but there may not be any such Ys, given Collapse.

4. "The ϕs"

The situation just encountered with "the ϕs" is worth a closer look. Philosophers who, following Boolos, have adopted irreducibly plural speech tend to use ordinary English plural terms of the form "the ϕs"—"the Cheerios in my bowl", "the sets", "the citizens of the United States", and so on—in addition to the rest of the apparatus of plural logic (plural quantifiers, variables, and the predicate 'is one of'). Given composition as identity, these need to be handled with care.

There are two ways to symbolize “the ϕs”. The first makes use of a plural definite description functor, I. Grammatically, I combines with a plural variable X and a formula Ψ to form a plural definite description $IX\Psi$, symbolizing “the Xs such that Ψ”. Semantically, $IX\Psi$ denotes the things that together satisfy Ψ. (If no Xs, or more than one Xs, together satisfy Ψ, then something has Gone Wrong.) I may be compared with the singular definite-description-forming functor ι, which combines with a singular variable x and a formula ψ to form the singular definite description $\iota x\psi$, a term that denotes the unique x that satisfies ψ (if any such x exists). The singular definite description $\iota x\psi$ may be read in ordinary English as “the ψ”, but beware of reading the plural definite description $IX\Psi$ as “the Ψs”. The term “the Ψs”, as it’s normally used in English, is intended to stand for things such that *each of them* is Ψ, whereas $IX\Psi$ stands for things that *collectively* are Ψ. (‘The Cheerios’ stands for things each of which is a Cheerio, not for things that collectively Cheerio.) The way to symbolize English terms of the form “the ϕs” (“the Cheerios”, “the sets”…) using I is this: $IX\forall y(Xy\longleftrightarrow\phi)$ (“the Xs that are such that something is one of them iff it is a ϕ”).

The other way to symbolize “the ϕs” is to first symbolize it using I and then to eliminate I using Russell’s theory of descriptions. Thus instead of saying “$IX\forall y(Xy\longleftrightarrow\phi)$ are Γ”, one may say instead “There are unique Xs such that $\forall y(Xy\longleftrightarrow\phi)$, and these Xs are Γ”.

On either way of symbolizing it, “the ϕs” does not behave as expected given Collapse. If there are no Xs such that something is one of them iff it is a ϕ, then $IX\forall y(Xy\longleftrightarrow\phi)$ has no denotation, and the Russellian symbolization of “the ϕs are Γ” comes out false for all Γ. And as we saw in the previous section, there don’t in general exist such Xs, given Collapse. There don’t, for example, exist things such that something is one of them iff it is human. “The humans” is an empty plural term.

Given composition as identity, then, we must be very careful with the locution “the ϕs”. To take one example: defenders of composition as identity often describe their view as implying that a person is identical to her subatomic particles. But given Collapse, the plural term ‘her subatomic particles’ denotes nothing. It is intended to denote Xs such that something is one of them iff it is one of the subatomic particles of the person in question; but any Xs including all subatomic particles of a person also include further things—things containing as parts multiple subatomic particles from the person.

5. McDaniel's argument

As an illustration of the moral of the previous section, consider Kris McDaniel's (2008) recent argument that composition as identity rules out strongly emergent properties.

Let a *naturalness isomorphism* be a 1-1 function that preserves both perfectly natural properties and relations and the part-whole relation; call w and z *duplicates* iff some naturalness isomorphism has domain $\{x|x < w\}$ and range $\{x|x < z\}$; and call the Xs and the Ys *plural duplicates* iff some naturalness isomorphism has domain $\{x|Xx\}$ and range $\{x|Yx\}$.[6] McDaniel begins by claiming that anyone who defends any form of composition as identity had better accept the following principle:

Plural duplication principle If w fuses the Xs, z fuses the Ys, and the Xs are plural duplicates of the Ys, then w and z are duplicates

For, McDaniel says, if the Xs are *collectively just like* the Ys, but w is *not* just like z, then, it would seem, either w or z has some intrinsic feature that pertains to it itself, invisible to anyone looking solely at its parts; and how could any such object be identical to its parts in any interesting sense? As McDaniel puts it, the plural duplication principle gives formal expression to the idea that "a full description of the parts is a full description of the whole" (p. 130).

The defender of the interesting and fun form of composition as identity we are considering here must indeed accept the plural duplication principle. For if w fuses the Xs and z fuses the Ys, then by Collapse, $\{x|x < w\} = \{x|Xx\}$ and $\{x|x < z\} = \{x|Yx\}$.

In the argument's second phase McDaniel argues that the plural duplication principle rules out *strongly emergent properties*—properties that do not "locally supervene on the perfectly natural properties and relations exemplified by only atomic material objects" (p. 131). Putative examples include the quantum states of entangled systems and qualitative properties of phenomenal experiences. In a third phase McDaniel goes on to say that strongly emergent properties are indeed possible, and perhaps even actual. But set aside the third phase—the defender of composition as identity can resist the second phase: the plural duplication principle does not rule out strongly emergent properties.[7]

[6] I've simplified and modified McDaniel's definitions a bit.

[7] Given the strong, fun, interesting version of composition as identity that I am considering here, anyway. My defense does not extend to the wimpy, dreary, boring forms of composition as identity defended by Lewis (1991) and me (2007).

Let F be a strongly emergent property. Here is the crucial passage:

> …Since F does not supervene on the perfectly natural properties and relations of the atomic parts of x, the ws, there could be some zs such that the zs are plural duplicates of the ws but the y that is composed of the zs does not exemplify F. (p. 131)

So x has F; some possible y does not have F; and:

(1) The atomic parts of x, the Ws, are plural duplicates of some Zs that y fuses

If (1) were true then we would indeed have a violation of the plural duplication principle. But (1) contains the problematic plural term 'the atomic parts of x'. It is supposed to refer to some Ws which are such that something is one of them iff it is an atomic part of x. But there are no such Ws. Any Ws including each atomic part of x will also include further things that are not atomic parts of x, namely composite things containing multiple atomic parts of x as parts (provided x has more than one atomic part, which it must if the example is to be coherent).

Why were we supposed to grant (1)? Because F is strongly emergent—i.e., does not "locally supervene on the perfectly natural properties and relations exemplified by only atomic material objects". But what this phrase surely means is the following (and the defender of composition as identity has no reason to admit strongly emergent properties under any stronger definition). Say that a property is *atomic* iff it never differs between a pair of objects x and y such that some naturalness isomorphism has domain $\{z|z < x \text{ and } z \text{ is atomic}\}$ and range $\{z|z < y \text{ and } z \text{ is atomic}\}$; the quoted phrase is surely intended to define strongly emergent properties as nonatomic ones—properties that do not supervene on the perfectly natural properties and relations distributed over the sets of their atomic parts. With strong emergence thus understood, all that is implied by F's being strongly emergent is that there could exist x and y where x has F, y does not, and the following claim (rather than (1)) holds:

(2) Some naturalness isomorphism has domain $\{z|z < x \text{ and } z \text{ is atomic}\}$ and range $\{z|z < y \text{ and } z \text{ is atomic}\}$

Unlike (1), (2) does not require the existence of a putative plurality of "the atomic parts of x". (2) speaks of the set, not the plurality, of atomic parts of x.

The role of (1) in the argument was to select some Ws that x fuses and are plural duplicates of some Zs that y fuses. It proposed Ws that include all and only atomic parts of x; but there are no such Ws. Are there any other Ws fitting the bill that the argument could utilize? No: no Ws that x fuses could be plural duplicates of any Zs that y fuses. In order for the Ws and the Zs to be plural duplicates, the set of things that are one of the Ws must be mapped one-to-one by some naturalness isomorphism, f, onto the set of things that are one of the Zs. But given Collapse, x itself is one of the Ws!—the Ws fuse to x and x is part of x. Moreover, since x is one of the Ws, f must map x to y (the argument for this is tedious but straightforward[8]); but x has the perfectly natural property F whereas y does not, which is incompatible with f being a naturalness isomorphism. Nor does (2)—which is all the defender of composition as identity who accepts strongly emergent properties is committed to—require saying otherwise. The naturalness isomorphism asserted to exist by (2) is defined only on the set of atomic parts of x, and do doesn't map x to anything; thus its existence is compatible with the fact that x and y differ over the property F.

Say that sets A and B are *set-duplicates* iff some naturalness isomorphism has domain A and range B; and let the "set duplication principle" say that the (schematic) fusions of (the members of) set-duplicates must be duplicates—i.e., if: sets A and B are set-duplicates, $w\,\text{S-Fu}_v v \in A$, and $z\,\text{S-Fu}_v v \in B$, then: w and z are duplicates. Unlike the plural duplication principle, the set-duplication principle does preclude strongly emergent properties (given (2), the sets of atomic parts of x and y above would be set-duplicates). But the defender of composition as identity is under no pressure to accept the set-duplication principle. She had to accept the plural duplication principle because she identifies an object o with some Xs whenever o Fu X; but she does *not* identify o with a set A whenever $o\,\text{S-Fu}_v v \in A$. Indeed, she could not, for this would lead to incompatible identifications: there are in general distinct sets A and B (corresponding to distinct decompositions of o) such that $o\,\text{S-Fu}_v v \in A$ and $o\,\text{S-Fu}_v v \in B$.

[8] Since f's range is $\{z|Zz\}$, $f(x)$ is one of the Zs, and so is part of the fusion of the Zs—i.e., y—by Collapse. So we have $f(x) < y$. Further, $f^{-1}(y)$ is one of the Ws; but x fuses the Ws; so $f^{-1}(y) < x$. But f preserves the part-whole relation; thus $f(f^{-1}(y))$—i.e., y—is part of $f(x)$. So by antisymmetry, $f(x) = y$.

6. Ideological simplifications

The following biconditional follows from the assumptions in section 1:

$$x{<}y \leftrightarrow \forall Y(Yy{\to}Yx) \tag{3}$$

For suppose $x < y$, and assume (i) Yy. Let z Fu Y (some such z exists given the plural fusions principle). By Collapse, (ii) $\forall w(Yw{\leftrightarrow}w{<}z)$. By (i) and (ii), $y < z$; so by transitivity, $x < z$; so by (ii), Yx. Conversely, suppose $\forall Y(Yy{\to}Yx)$. By comprehension, there exist some Xs and z such that (i) z Fu_v $v{=}y$ and (ii) $\forall w(Xw{\leftrightarrow}w{<}z)$. By (i), $y < z$; so by (ii), Xy; so by the supposition, Xx; so by (ii), $x < z$; so by (i) and tedious mereology, $x < y$. (Every part of x overlaps y (if $w < x$ then since $x < z$, $w < z$; whence by (i), Owy). So by strong supplementation, $x < y$.)

So any defender of composition as identity already agrees that any formula $x{<}y$ attributing parthood is equivalent, relative to her doctrine, to some formula that contains neither $<$ nor anything defined in terms of it, namely: $\forall Y(Yy{\to}Yx)$. This suggests the possibility of taking the equivalence (3) as a definition of $<$, thus eliminating $<$ from fundamental ideology.

If this is done, all the old assumptions governing the previously primitive $<$ (mereology, comprehension, and composition as identity itself) must now be taken as assumptions governing the remaining primitive notions, in particular identity and 'is one of'. For example, the assumption that $<$ is transitive now becomes, when rewritten using (3): $\forall x\forall y\forall z((\forall Y(Yy{\to}Yx)\wedge\forall Y(Yz{\to}Yy))\to\forall Y(Yz{\to}Yx))$. A very simple law of mereology has become a much more complex assumption about the behavior of the remaining primitive notions. Similar remarks apply to the other old assumptions. These increases in complexity of doctrine must be weighed against the decreased ideological complexity.

There is room for further ideological simplification. Given composition as identity and the other background assumptions in place, it's easy to show that these three biconditionals also hold:

$$\begin{aligned}
&\alpha=\beta \leftrightarrow \alpha{<}\beta \wedge \beta{<}\alpha && \text{if } \alpha \text{ and } \beta \text{ are singular}\\
&\alpha=\beta \leftrightarrow \forall x(\alpha x{\leftrightarrow}\beta x) && \text{if } \alpha \text{ and } \beta \text{ are plural}\\
&\alpha=\beta \leftrightarrow \beta\alpha \wedge \forall x(\beta x{\to}x{<}\alpha) && \text{if } \alpha \text{ is singular and } \beta \text{ is plural}
\end{aligned} \tag{4}$$

Since $<$ can then be eliminated using (3), one could use (4) to eliminate $=$ as well, provided, as before, that the old logical assumptions governing $=$ are rewritten as assumptions governing the remaining primitives. Further, one can also use Leibniz's Law (remember the expanded grammar) to demonstrate:

$$\forall X \phi \leftrightarrow \forall x \phi[X/x] \tag{5}$$

where $\phi[X/x]$ is the result of substituting free xs for free Xs in ϕ. (The important facts are that each x is identical to the Xs that are x's parts, and any Xs are identical to the x that is their fusion.) One could then use (5) to eliminate the plural quantifier from primitive ideology.

So the defender of composition as identity can accept an ideology that is a sort of hybrid of first-order and plural logic. It contains just one sort of variable—that is, there is no distinction between singular and plural variables—but it also contains 'is one of'. Identity and parthood are defined, not primitive. But it's hard to see how this ideology is simpler than that of a standard first-order mereological theory in which the symbol for parthood is written as "is one of", and in which $x = y$ is defined as meaning $x<y \wedge y<x$. Moreover, the theory's basic mereological and logical laws are highly complex, since they result from using (3)–(5) to eliminate $<$, $=$, and the plural quantifiers from the much simpler standard laws. The theory is unattractive.

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