Consequences of collapse*

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Composition as identity is the strange and strangely compelling doctrine that the whole is in some sense identical to its parts. According to the most interesting and fun version, the one inspired\(^1\) by Donald Baxter, this is meant in the most straightforward way: a single whole is genuinely identical to its many parts taken together—identical in the very same sense of ‘identical’, familiar to philosophers, logicians, and mathematicians, in which I am identical to myself and \(2 + 2\) is identical to \(4\).

Composition as identity implies the principle of Collapse: something is one of the \(X\)s iff it is part of the fusion of the \(X\)s. (Collapse is so-called because it in effect identifies mereologically equivalent pluralities.) In an earlier paper I pointed out that Collapse alters Boolos’s logic of plural quantification in various ways.\(^2\) Here I point out some further consequences of Collapse. For example, collapse implies that plural definite descriptions do not function normally. (As we will see, this undermines Kris McDaniel’s (2008) recent argument against composition as identity.) Also it opens the door to drastic—though arguably unattractive—ideological simplifications: parthood, identity, and the plural quantifiers may all be eliminated.

1. Composition as identity formulated

Composition as identity is a logically radical thesis, since it holds that a single thing can be identical to many things. In order to state this thesis, we need a nonstandard logical language.

The nonstandard language includes the primitive notions of first order logic, plus plural quantifiers and variables (symbolize “for some \(X\)s” as \(\exists X\)),

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\(^1\)Baxter (1988a, b). I say “inspired” because Baxter’s own view is even more radical; see Baxter (2014).

\(^2\)Sider (2007). On that basis I rejected (fun, interesting) composition as identity, but I no longer find that argument convincing since I now doubt that Boolos’s logic should be taken as metaphysically basic (Sider, 2011, section 9.15).
plus ‘y is one of the Xs’ (symbolized: Xy), plus a predicate for parthood, <, plus an identity predicate =. What is nonstandard is that in identity predications, each flanking variable may be either plural or singular. Thus x = y, x = Y, Y = x, and X = Y are all grammatical. Define overlap and fusion thus:

\[ O_{xy} =_{df} \exists z(z < x \land z < y) \]
\[ x Fu Y =_{df} \forall z(Yz \rightarrow z < x) \land \forall z(z < x \rightarrow \exists w(Yw \land Ozw)) \]

(Objects overlap when they share a part in common; x is a fusion of the Ys iff anything that is one of the Ys is part of x, and each part of x overlaps something that is one of the Ys.) Composition as identity may then be formulated as follows:

\[ \forall x \forall Y(x Fu Y \rightarrow x = Y) \]

(Composition as identity)

I will take this core claim of composition as identity to be accompanied by some further assumptions. First: classical first-order mereology, including the usual principles of reflexivity, transitivity, antisymmetry, strong supplementation, and so forth, plus the following fusions principle:

\[ \forall Y \exists x \ x Fu Y \] (Fusions)

Second:

\[ \alpha = \beta, \psi(\alpha) \vdash \psi(\beta) \] (Leibniz’s Law)

(where \( \psi(\alpha) \) and \( \psi(\beta) \) differ by exchanging zero or more occurrences of \( \alpha \) for \( \beta \) or \( \beta \) for \( \alpha \). And third:

\[ \forall x \forall z(x < z \rightarrow \exists Y(z Fu Y \land Yx)) \] (Plural covering)

Leibniz’s Law is intended to apply to all terms \( \alpha \) and \( \beta \), singular or plural—including the case where one of \( \alpha \) and \( \beta \) is singular and the other is plural. This requires an even more grammatically nonstandard language than was indicated above. When \( y = X \), the law says that each of \( \psi(y) \) and \( \psi(X) \) implies the other, and so the grammar must allow any predicate positions in \( \psi \) that can

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3I argued in Sider (2007, section 3.2) that composition as identity implies all of classical mereology except for unrestricted composition, but my arguments employed a plural logic with a primitive plural-term-forming operator ‘and’. I don’t see how to make analogous arguments in the present context, even after the addition of weakened comprehension (section 3).
be occupied by \( y \) to be occupied by \( X \), and any predicate positions that can be occupied by \( X \) to be occupied by \( y \). Thus predicate positions cannot be fixedly singular or plural. So in particular, \(<\) must be allowed to take plural variables on either side, and ‘is one of’ must be allowed to take plural variables on its left and singular variables on its right. If the whole, \( y \), is genuinely identical to its many parts, the \( X \)s, then how could the \( X \)s fail to be part of \( z \), or one of the \( Z \)s, if \( y \) is part of \( z \) and one of the \( Z \)s?\(^4\)

Plural covering says that if \( x \) is part of \( z \) then \( x \) is one of some \( Y \)s whose fusion is \( z \). In most developments of mereology and plural logic, this sort of principle would be derived from the principles of mereology plus a plural comprehension principle (the \( Y \)s could be taken to be “\( x \) and \( z \)”, i.e., the things such that \( w \) is one of them iff \( w = x \) or \( w = z \)), but as we’ll see in section 3, composition as identity creates problems for the usual form of plural comprehension.

2. Collapse

Composition as identity, together with the assumptions listed in the previous section, implies Collapse:\(^5\)

\[
\forall X \forall z (z \text{ Fu } X \rightarrow \forall y (X y \leftrightarrow y < z)) \tag{Collapse}
\]

(“\( y \) is one of the \( X \)s iff \( y \) is part of the fusion of the \( X \)s”). For suppose \( z \text{ Fu } X \), and take any \( y \). Suppose first that \( X y \); then by the definition of ‘Fu’, \( y < z \). Conversely, suppose \( y < z \). By plural covering, for some \( Y \), \( z \text{ Fu } Y \) and \( Y y \). Since \( z \text{ Fu } X \) and \( z \text{ Fu } Y \), by composition as identity, \( z = X \) and \( z = Y \); and hence, by Leibniz’s Law, \( X = Y \); and since \( Y y \), by Leibniz’s Law, \( X y \).

Given Collapse, there are fewer pluralities than one normally expects. There are, for example, no \( X \)s such that something is one of them if and only if it is a human being. For any \( X \)s including all humans will also include some nonhumans, and thus will not include only humans. If each human is one of the \( X \)s then the fusion of the \( X \)s (which must exist given the fusions principle) contains many non-human parts (nonhuman parts of individual humans, and nonhuman objects containing multiple humans as parts, for example), and each non-human part of the fusion of the \( X \)s must be one of the \( X \)s given Collapse. More generally, there will not exist pluralities that include all and only \( F \)s,

\(^4\)See Sider (2007, section 3.1).
\(^5\)Sider (2007, section 3.2) argues for this conclusion in a different formal setting.
except when, roughly speaking, each part of the aggregate of the $F$s is itself an $F$.\textsuperscript{6}

3. Comprehension

First consequence of Collapse: the comprehension principle for plural logic needs to be weakened. Boolos’s plural language is usually supposed to have a logic analogous to monadic impredicative second-order logic (minus the empty plurality), and thus to obey the following schema:

$$\exists x \phi \rightarrow \exists Y \forall x (Y x \leftrightarrow \phi)$$

(Comprehension)

(“provided there's at least one $\phi$, there are some things such that something is one of them iff it is a $\phi$”). But Comprehension fails given Collapse. For as we just saw, Collapse prohibits there being things such that something is one of them iff it is a human being.

So defenders of composition as identity must weaken Comprehension in some way. Comprehension guarantees the existence of a plurality corresponding to any given nonempty condition; a natural weakening restricts the guarantee to conditions that are “fusion-closed” in the sense that, roughly, they are satisfied by $x$ iff $x$ is part of the fusion of all things satisfying the condition.

But what, exactly, is a fusion-closed condition? The natural first answer is: a condition $\phi$ such that $\phi(x)$ iff $x$ is part of the fusion of the $\phi$s. But what are “the $\phi$s”? Some $Y$s such that something is one of them iff it is a $\phi$, presumably; but as we have seen, there will not in general be such $Y$s, given Collapse.

To solve this problem, we need to introduce a second notion of fusion. Let $\phi$ be any formula, $v$ any variable, and $\phi_v(v')$ the result of changing free $v$s to free $v'$s in $\phi$. The notion of an S-fusion (S for schematic) may then be defined as follows:

$$x \text{ S-Fu}_v \phi \equiv_{df} \forall z (\phi_v(z) \rightarrow z < x) \land \forall z (z < x \rightarrow \exists w (\phi_v(w) \land Ozw))$$

($x$ is an S-fusion of the $\phi$s iff each $\phi$ is part of $x$ and each part of $x$ overlaps some $\phi$). S-fusion is “schematic” because the variables $\phi$ and $v$ in the definition are not in the object language, but are rather metalinguistic; the definition supplies a definiens whenever $\phi$ and $v$ are replaced with a formula and variable.

\textsuperscript{6}More carefully: when each part of the $x$ such that $x \text{ S-Fu}_v F v$ is an $F$ (see section 3).
of the object language. I’ll assume that composition as identity is additionally accompanied by the following assumption:7

$$\exists y \phi_v(y) \rightarrow \exists y \ S-Fu_v \phi$$  \quad \text{(Fusions, schematic form)}

We can now define a fusion-closed condition as a condition that is satisfied by an object if and only if that object is part of the S-fusion of the condition. And we can state the weakened form of comprehension that we were after:

$$\exists x \phi_v(x) \rightarrow \exists Y \exists z (z \ S-Fu_v \phi \land \forall x (Y x \leftrightarrow x < z))$$  \quad \text{(Weak comprehension)}

(“provided there’s at least one \( \phi \), there are some things such that something is one of them iff it is part of the S-fusion of \( \phi \)’s”). This is in effect a restriction of the usual form of comprehension to fusion-closed conditions.

Unlike the usual form of comprehension, weak comprehension does not conflict with Collapse. But it does have some of the implications that the usual form has. For instance, it can be used to derive plural covering. For let \( x < z \). By weak comprehension, for some \( Y \)'s and some \( o \), (i) \( o \ S-Fu_v (v=x \lor v=z) \) and (ii) \( \forall w (Y w \leftrightarrow w < o) \). By tedious mereology, \( o = z \). (By (i), \( z < o \). Also, by (i), every part of \( o \) overlaps either \( z \) or \( x \), and so overlaps \( z \); so \( o < z \) by strong supplementation. So by antisymmetry, \( o = z \).) Then, by further tedious mereology, \( z \ Fu Y \). (Take any \( w \) such that \( Y w \); then by (ii), \( w < o \); so, \( w < z \). Next, take any \( w \) such that \( w < z \); then \( w < o \); so by (ii), \( Y w \); so \( w \) overlaps one of the \( Y \)'s—namely, itself.) And since \( x < z \), \( x < o \), and so by (ii), \( Y x \).

4. “The \( \phi \)'s”

The situation encountered in the previous section with “the \( \phi \)'s” is worth a closer look. Philosophers who, following Boolos, have adopted irreducibly plural speech tend to use ordinary English plural terms of the form “the \( \phi \)'s”—“the Cheerios in my bowl”, “the sets”, “the citizens of the United States”, and so on—in addition to the rest of the apparatus of plural logic (plural quantifiers, variables, and the predicate ‘is one of’). Given composition as identity, these plural terms need to be handled with care.

There are two ways to symbolize “the \( \phi \)'s”. The first makes use of a plural definite description functor, \( I \). Grammatically, \( I \) combines with a plural variable

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7 The schematic fusions principle implies the original fusions principle, for we may set \( \phi \) to \( X v \). (I allow free variables in instances of the schematic fusions principle.) Indeed, we could define “\( x \ Fu Y \)” as meaning “\( x \ S-Fu_v \ Y v \)”.
$X$ and a formula $\Psi$ to form a plural definite description $IX\Psi$, symbolizing “the $X$s such that $\Psi$”. Semantically, $IX\Psi$ denotes the things that together satisfy $\Psi$. (If no $X$s, or more than one $X$s, together satisfy $\Psi$, then something has Gone Wrong.) $I$ may be compared with the singular definite-description-forming functor $\iota$, which combines with a singular variable $x$ and a formula $\phi$ to form the singular definite description $\iota x \phi$, a term that denotes the unique $x$ that satisfies $\phi$ (if any such $x$ exists). The singular definite description $\iota x \phi$ may be read in ordinary English as “the $\phi$”, but beware of reading the plural definite description $IX\Psi$ as “the $\Psi$s”. The term “the $\Psi$s”, as it’s normally used in English, is intended to stand for things such that each of them is $\Psi$, whereas $IX\Psi$ stands for things that collectively are $\Psi$. (‘The Cheerios’ stands for things each of which is a Cheerio, not for things that collectively Cheerio.) The way to symbolize English terms of the form “the $\phi$s” (“the Cheerios”, “the sets”...) using $I$ is this: $IX\forall y(Xy \leftrightarrow \phi)$ (“the $X$s that are such that something is one of them iff it is a $\phi$”).

The other way to symbolize “the $\phi$s” is to first symbolize it using $I$ and then to eliminate $I$ using Russell’s theory of descriptions. Thus instead of saying “$IX\forall y(Xy \leftrightarrow \phi)$ are $\Gamma$”, one may say instead “There are unique $X$s such that $\forall y(Xy \leftrightarrow \phi)$, and these $X$s are $\Gamma$”.

On either way of symbolizing it, “the $\phi$s” does not behave as expected given Collapse. If there are no $X$s such that something is one of them iff it is a $\phi$, then $IX\forall y(Xy \leftrightarrow \phi)$ has no denotation, and the Russelian symbolization of “the $\phi$s are $\Gamma$” comes out false for all $\Gamma$. And as we saw in the previous section, there don’t in general exist such $X$s, given Collapse. There don’t, for example, exist things such that something is one of them iff it is human. “The humans” is an empty plural term.

Given composition as identity, then, we must be very careful with the locution “the $\phi$s”. To take one example: defenders of composition as identity often describe their view as implying that a person is identical to her subatomic particles. But given Collapse, the plural term ‘her subatomic particles’ denotes nothing. It is intended to denote $X$s such that something is one of them iff it is a subatomic particle that is part of the person in question; but any $X$s of which each such part of a person is one will also include further things—anything (such as the person’s head) that contains multiple subatomic particles from the person will also be one of such $X$s.
5. McDaniel’s argument

As an illustration of the moral of the previous section, consider Kris McDaniel’s (2008) argument that composition as identity rules out strongly emergent properties.

Let a *naturalness isomorphism* be a 1-1 function that preserves both perfectly natural properties and relations and the part-whole relation; call \( w \) and \( z \) *duplicates* iff some naturalness isomorphism has domain \( \{ x \mid x < w \} \) and range \( \{ x \mid x < z \} \); and call the \( X \)s and the \( Y \)s *plural duplicates* iff some naturalness isomorphism has domain \( \{ x \mid Xx \} \) and range \( \{ x \mid Yx \} \).\(^8\) McDaniel begins by claiming that anyone who defends any form of composition as identity had better accept the following principle:

**Plural duplication principle** If \( w \) fuses the \( X \)s, \( z \) fuses the \( Y \)s, and the \( X \)s are plural duplicates of the \( Y \)s, then \( w \) and \( z \) are duplicates.

For, McDaniel says, if the \( X \)s are *collectively just like* the \( Y \)s, but \( w \) is *not* just like \( z \), then, it would seem, either \( w \) or \( z \) has some intrinsic feature that pertains to it itself, invisible to anyone looking solely at its parts; and how could any such object be identical to its parts in any interesting sense? As McDaniel puts it, the plural duplication principle gives formal expression to the idea that “a full description of the parts is a full description of the whole” (p. 130).

The defender of the interesting and fun form of composition as identity must indeed accept the plural duplication principle. For if \( w \) fuses the \( X \)s and \( z \) fuses the \( Y \)s, then by Collapse, \( \{ x \mid x < w \} = \{ x \mid Xx \} \) and \( \{ x \mid x < z \} = \{ x \mid Yx \} \).

In the argument’s second phase McDaniel argues that the plural duplication principle rules out *strongly emergent properties*—properties that do not “locally supervene on the perfectly natural properties and relations exemplified by only atomic material objects” (p. 131). Putative examples include the quantum states of entangled systems and qualitative properties of phenomenal experiences. In a third phase McDaniel goes on to say that strongly emergent properties are indeed possible, and perhaps even actual. But set aside the third phase—the defender of composition as identity can resist the second phase: the plural duplication principle does not rule out strongly emergent properties.\(^9\)

Let \( F \) be a strongly emergent property. Here is the crucial passage:

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\(^8\)I've simplified and modified McDaniel's definitions a bit.

\(^9\)Given the strong, fun, interesting version of composition as identity, anyway. My defense does not extend to the wimpy, dreary, boring forms of composition as identity defended by Lewis (1991) and me (2007).
...Since \( F \) does not supervene on the perfectly natural properties and relations of the atomic parts of \( x \), the \( ws \), there could be some \( zs \) such that the \( zs \) are plural duplicates of the \( ws \) but the \( y \) that is composed of the \( zs \) does not exemplify \( F \). (McDaniel, 2008, p. 131)

So \( x \) has \( F \); some possible \( y \) does not have \( F \); and:

(1) The atomic parts of \( x \), the \( Ws \), are plural duplicates of some \( Zs \) that \( y \) fuses

If (1) were true then we would indeed have a violation of the plural duplication principle. But (1) contains the problematic plural term ‘the atomic parts of \( x \)’. It is supposed to refer to some \( Ws \) which are such that something is one of them iff it is an atomic part of \( x \). But there are no such \( Ws \). Given Collapse, something is one of the \( Ws \) iff it is part of the fusion of the \( Ws \). So any \( Ws \) including each atomic part of \( x \) will also include further things that are not atomic parts of \( x \), namely, composite things containing multiple atomic parts of \( x \) as parts (provided \( x \) has more than one atomic part, which it must if the example is to be coherent).

Why were we supposed to grant (1)? Because \( F \) is strongly emergent—i.e., does not “locally supervene on the perfectly natural properties and relations exemplified by only atomic material objects”. But what this phrase surely means is the following (and the defender of composition as identity has no reason to admit strongly emergent properties under any stronger definition). Say that a property is \textit{atomically supervenient} iff it never differs between a pair of objects \( x \) and \( y \) such that some naturalness isomorphism has domain \{\( z | z < x \) and \( z \) is atomic\} and range \{\( z | z < y \) and \( z \) is atomic\}; the quoted phrase is surely intended to define strongly emergent properties as properties that are not atomically supervenient—properties that do not supervene on the perfectly natural properties and relations distributed over the sets of their atomic parts. With strong emergence thus understood, all that is implied by \( F \)’s being strongly emergent is that there could exist \( x \) and \( y \) where \( x \) has \( F \), \( y \) does not, and the following claim (rather than (1)) holds:

(2) Some naturalness isomorphism has domain \{\( z | z < x \) and \( z \) is atomic\} and range \{\( z | z < y \) and \( z \) is atomic\}

Unlike (1), (2) does not require the existence of a putative plurality of “the atomic parts of \( x \)”. (2) speaks of the set, not the plurality, of atomic parts of \( x \).
The role of (1) in the argument was to select some Ws that x fuses and are plural duplicates of some Zs that y fuses. It proposed Ws that include all and only atomic parts of x; but there are no such Ws. Are there any other Ws fitting the bill that the argument could utilize? No: no Ws that x fuses could be plural duplicates of any Zs that y fuses. In order for the Ws and the Zs to be plural duplicates, the set of things that are one of the Ws must be mapped one-to-one by some naturalness isomorphism, f, onto the set of things that are one of the Zs. But given Collapse, x itself is one of the Ws!—the Ws fuse to x and x is part of x. Moreover, since x is one of the Ws, f must map x to y (the argument for this is tedious but straightforward\(^1\)); but x has the perfectly natural property F whereas y does not, which is incompatible with f being a naturalness isomorphism. Nor does (2)—which is all the defender of composition as identity who accepts strongly emergent properties is committed to—require saying otherwise. The naturalness isomorphism asserted to exist by (2) is defined only on the set of atomic parts of x, and so doesn’t map x to anything; thus its existence is compatible with the fact that x and y differ over the property F.

Say that sets A and B are set duplicates iff some naturalness isomorphism has domain A and range B. And let the “set duplication principle” say that the fusions of set duplicates must themselves be duplicates. More carefully, in terms of the notion of schematic fusion from section 3: if A and B are set duplicates, if w S-Fu v ∈ A, and if z S-Fu v ∈ B, then w and z are duplicates. Unlike the plural duplication principle, the set duplication principle does preclude strongly emergent properties (given (2), the sets of atomic parts of x and y above would be set duplicates). But the defender of composition as identity is under no pressure to accept the set-duplication principle. She had to accept the plural duplication principle because she identifies an object o with some Xs whenever o Fu X; but she does not identify o with a set A whenever o S-Fu v ∈ A. Indeed, she could not, for this would lead to incompatible identifications: there are in general distinct sets A and B (corresponding to distinct decompositions of o) such that o S-Fu v ∈ A and o S-Fu v ∈ B.

\(^1\)Since f’s range is \{z | Z z\}, f(x) is one of the Zs, and so is part of the fusion of the Zs—i.e., y—by Collapse. So we have f(x) < y. Further, f\(^{-1}\)(y) is one of the Ws; but x fuses the Ws; so f\(^{-1}\)(y) < x. But f preserves the part-whole relation; thus f(f\(^{-1}\)(y))—i.e., y—is part of f(x). So by antisymmetry, f(x) = y.
6. Ideological simplifications

Composition as identity (together with the accompanying assumptions mentioned in section 1) in effect collapses the plural/singular distinction, by implying the following claims:

\[ \forall x \exists Y x = Y \]
\[ \forall X \exists y X = y \]  

(plural/singular collapse)

To establish the first, reflexivity yields \( x < x \); by plural covering, \( x \) is a fusion of some \( Y \)'s; \( x \) is then identical to those \( Y \)'s by composition as identity. To establish the second, note that some \( y \) is a fusion of the \( X \)'s by the fusions principle, and is then identical to them by composition as identity.

Given plural/singular collapse, we may establish each of the following schemas, where \( \alpha \) and \( \beta \) may be any terms, plural or singular:

\[ \alpha < \beta \iff \beta \alpha \]  

(<)

\[ \alpha = \beta \iff (\alpha \beta \wedge \beta \alpha) \]  

(=)

\[ \forall X \phi_v(X) \iff \forall x \phi_v(x) \]  

(\( \forall X \))

That is, \( \alpha \) is part of \( \beta \) iff \( \alpha \) is one of \( \beta \), \( \alpha = \beta \) iff \( \alpha \) is one of \( \beta \) and \( \beta \) is one of \( \alpha \), and every plurality \( \phi \) (or better, “all thingses \( \phi \)”) iff everything \( \phi \)'s. (Instances of these schemas are indeed grammatical—remember the nonstandard grammar introduced in section 1 to allow the strong form of Leibniz’s Law.)

To establish (<), begin by noting that for some \( a \) and \( B \), \( a = \alpha \) and \( B = \beta \). (This is trivial when \( \alpha \) is singular and \( \beta \) is plural, and otherwise follows from plural/singular collapse; ‘\( a \)' here is a singular variable—as is ‘\( b \)' below—and ‘\( B \)' is a plural variable.) By the fusions principle, for some \( z \), \( z Fu B \). By Collapse, \( a < z \) iff \( Ba \); by composition as identity, \( z = B \); and then by Leibniz’s Law, (<) follows. (=): for some \( a \) and \( b \), \( a = \alpha \) and \( b = \beta \). By antisymmetry and reflexivity, \( a = b \iff (a < b \wedge b < a) \); (=) then follows by (<) and Leibniz’s Law. (\( \forall X \)): suppose \( \forall X \phi_v(X) \), and consider any \( x \). By plural/singular collapse, \( x = Y \) for some \( Y \); by the supposition, \( \phi_v(Y) \), and then by Leibniz’s Law, \( \phi_v(x) \). Suppose next that \( \forall x \phi_v(x) \), and consider any \( X \)'s. By plural/singular collapse, \( X = y \) for some \( y \); by the supposition, \( \phi_v(y) \), and so by Leibniz’s Law, \( \phi_v(X) \).

Each of these schemas gives the defender of composition an option for simplifying the ideology of her theory. (<) provides the option of eliminating
<, the predicate for parthood, in favor of ‘is one of’ (the latter is symbolized, recall, by concatenating terms). For consider the result of “translating” the theory using (<): replace each formula α < β occurring in the theory, whether standing alone or within a larger subformula, with the < -free formula β α. Given (<) and the principle of substitution of equivalents, this translation procedure preserves truth value. So there is no extensional obstacle, anyway, to replacing the original theory of composition as identity with this translation. Similarly, (=) provides the option of eliminating the identity sign in favor of ‘is one of’, and (∀ X) together with the equivalence of ∃ X with ∼ ∀ X ∼ provide the option of eliminating plural in favor of singular quantification.

The eliminations using (<) and (∀ X) can each be reversed: (<) could be used to eliminate ‘is one of’ in favor of <, and (∀ X) could be used to eliminate singular in favor of plural quantification. Moreover, these eliminations can be combined. For instance, one could use (<), (=), and (∀ X) to eliminate <, =, and plural quantification in favor of ‘is one of’ and singular quantification (again, remember the nonstandard grammar).

Though they may be initially tempting, on further scrutiny these ideological simplifications are not advisable for the defender of composition as identity (nor does their availability make that theory more attractive). Although the translations using (<), (=), and (∀ X) do not alter truth value, it would be natural to regard them as changing the content of the theory. For example, even if it makes no difference to the truth value of a statement whether its quantifiers and variables are singular or plural (as (∀ X) says), the defender of composition as identity might reasonably insist that there is a difference between quantifying plurally and singularly—between there being some things that φ and there being a single thing that φ’s. Or consider again the result of doing all three translations: of eliminating <, =, and plural quantification and variables in favor of ‘is one of’ and singular quantification. It is hard not to regard the resulting theory as a terminological variant of a standard first-order mereological theory without identity—i.e., one not based on composition as identity.\(^\text{11}\) The resulting theory’s predicate ‘is one of’ seems no different in content from the standard mereological theory’s predicate for parthood—it attaches to singular variables and obeys the assumptions of standard mereology—and the resulting theory has no other notions with which to say anything distinctive about parthood. Thus the “ideological simplification” seems to have obliterated the intuitive content of composition as identity. To capture that intuitive content, one needs distinct

\(^{11}\) x = y in such a language can be defined as meaning x < y ∧ y < x.
notions of parthood, ‘is one of’, and singular and plural quantification.\textsuperscript{12}

References


\\textsuperscript{12}Furthermore, any set of axioms for the original theory will become more complex when \(=\) is eliminated via (=). (Of course, there \textit{might} be some simpler axiomatization of the resulting theory other than the result of translating axioms of the old theory, but there is no guarantee of this.) The issues here are difficult, but my own view is that simplicity of the fundamental laws governing a theory’s fundamental ideology, in addition to the simplicity of that ideology itself, is relevant to theory choice in fundamental metaphysics (see Sider (2013, section 1)).