STRUCTURALISM

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Structuralist slogan: "Only structure matters"

1. Set theoretic platonism and structuralism

Set theoretic platonism partially fits this slogan, since sets deserve to be defined as mathematical objects only because of their structural features.

> (Thus multiple set-theoretic definitions would be adequate. E.g. we could reverse $\varnothing$ and $\{\varnothing\}$ in the sequence of natural numbers.)

> (Thus perhaps ordinary talk of natural numbers is massively ambiguous.)

But sets themselves don't fit the slogan. We are talking about particular entities when we say that set A is a member of set B.

2. Ante rem structuralism

To more closely adhere to the slogan, ante rem structuralists posit new entities, *structures* and *positions in structures*.

(A baseball defense is a kind of structure, containing the positions of shortstop, center field, etc. This structure and positions exist independently of being exemplified by particular baseball players.)

Ante rem structuralists say that mathematical objects *are* positions in structures. E.g., the number 0 is the first position in the *natural number structure*—the structure that is exemplified by any sequence of entities with the same structure as the natural numbers.

Objection: the sequence of positions in the natural number structure:

first position   *second position*   *third position*   …

is just one sequence among many with the right structure to count as the natural numbers. So given the slogan, why should it alone constitute the true natural number sequence?

3. An unsolvable problem?

Any view about the foundations of mathematics will put forward particular entities structured in some particular way to undergird mathematics; so it seems that no such view could comply with the slogan.

Possible solution: the slogan is about methodology, not metaphysics. Mathematics only *cares* about structure, but more might be *true* beyond structure.