Linnebo on thin existence

Ted Sider
Philosophy of Mathematics

1. The idea of thin objects

> Are there objects that are "thin" in the sense that their existence does not make a substantial demand on the world? Frege famously thought so. He claimed that the equinumerosity of the knives and the forks on a properly set table suffices for there to be objects such as the number of knives and the number of forks, and for these objects to be identical. Versions of the idea of thin objects have been defended by contemporary philosophers as well. For example, Bob Hale and Crispin Wright assert that
>
> > what it takes for "the number of Fs = the number of Gs" to be true is exactly what it takes for the Fs to be equinumerous with the Gs, no more, no less. […] There is no gap for metaphysics to plug.
>
> (Linnebo 2018, p. xi; Hale and Wright 2009, p. 193)

Epistemic significance of this idea:

> The vast ontology of mathematics may well be problematic when understood in a thick sense. If mathematical objects are understood on the model of, say, elementary particles, there would indeed be good reason to worry about epistemic access and ontological extravagance. But this understanding of mathematical objects is not obligatory. If there are such things as thin objects, then the existence of mathematical objects need not make much of a demand on the world. It may, for instance, suffice that the theory purporting to describe the relevant mathematical objects is coherent. This would greatly simplify the problem of epistemic access. Although our knowledge of the coherence of mathematical theories is still inadequately understood, it is at least not a complete mystery in the way that knowledge of thick mathematical objects would be. More generally, the less of a demand the existence of mathematical objects makes on the world, the easier it will be to know that the demand is satisfied. (Linnebo, 2018)

Metaphysical significance:

> If little or nothing is required for the existence of objects of some sort, then no wonder there is an abundance of such objects. The less that is

> required for the existence of certain objects, the more such objects there will be. Thus, if mathematical objects are thin, this will explain the striking fact that mathematics operates with an ontology that is far more abundant than that of any other science. (Linnebo, 2018, p. xi)

1.1 Does thinness help with epistemology?

Given the truth of the doctrine of metaphysics of thin objects, how does its epistemology relate to the epistemology of mathematics?

If the puzzles about mathematical knowledge were specific to objecthood, the doctrine might be easier to know.

Suppose that the doctrine says that P is a sufficient basis for mathematical statement M. One might say given the mere truth of the doctrine, knowing P suffices for knowing M. But this sort of externalism might explain mathematical knowledge all on its own.

Related point: Linnebo says that mathematical existence claims aren't "risky". But the doctrine itself is risky (metaphysically, not ontically).

Thomasson objects to this kind of thought:

> …imagine that you are taking a two-year-old to the zoo, and approaching the giraffes. The child says: 'Can we go see the elephant now?' You reply: 'We'll see that after'. 'The after! I want to see that after! Pick me up now so I can see the after!'. Now you need to correct the child's misunderstanding—'No, I didn't mean to say we could see an after …' you begin. 'What!' the (curiously precocious) child responds. 'Are you saying that of all the creatures in the world, none are afters? Why, you're making a substantive biological claim about the kinds of creatures there are and aren't—but you're not even a biologist, and surely you haven't done the research to know that, of all the kinds of animal in the world, none are afters!' This accusation, of course, would be misguided. To correct the child's mistake, and to refrain from endorsing the idea that there are creatures that are afters, you needn't be making a substantive claim about the kinds of creature there are (and are not). Instead, you might justly say, you don't even know what it would mean for there to be an after-creature; the idea that there is an after-creature does not even make sense, given the role of the word 'after'. You need only be pointing out that a mistake has been made about the role of a term like 'after': that it is to mark an ordering of events (we'll go see the elephant after we see

> the giraffes), not a term attempting to name a sort of creature. And to point this out, we needn't be making a substantive biological claim about the sorts of creatures there are and aren't, but only about the different functions of different pieces of language. (Thomasson, 2015, 307–8)

1.2 Does thinness explain how many mathematical objects there are?

2. The asymmetric picture

(Dir) For any lines, l_1 and l_2 the direction of l_1 = the direction of l_2 iff l_1 is parallel to l_2

For such abstraction principles, Linnebo thinks of the right-hand-side as being sufficient for the left, in a sense that he formalizes using a grounding-like operator: "l_1 is parallel to $l_2 \Rightarrow$ the direction of l_1 = the direction of l_2.

> Might there be an easy way to obtain the desired sufficiency statements? Perhaps the right-hand side is merely a fancy—and syntactically misleading—rewording of the left-hand side. This proposal would certainly ensure that the right-hand side demands no more of the world than the left-hand side. Whatever its other merits, the proposal is clearly inadequate for our purposes. If the right-hand side is merely a façon de parler for the left-hand side, this would at most justify speaking as if there are abstract objects. But our aim is to make sense of how there in fact are such objects and of how we come to know about them. For a sufficiency claim to have this philosophical payoff, both sentences must be taken at face value. Every singular term must function as such semantically; that is, it must be in the business of referring to an object. And there must be no singular reference other than what is effected by the singular terms that occur in the relevant sentence. These considerations motivate the following constraint: (Linnebo, 2018, p. 15)

Face value constraint The formulas involved in a sufficiency statement $\phi \Rightarrow \psi$ can be taken at face value in our semantic analysis.

"Face value": singular terms on the right-hand-side refer to objects.

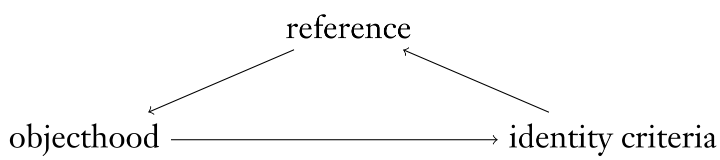
> –which quantifier-meaning is used here?

> The competing asymmetric picture denies that the two sides of a legitimate abstraction principle are on a par in every worldly respect. Instead, abstraction is regarded as an inherently asymmetric matter, where abstrac-

> tion on "old" entities gives rise to "new" objects. While the left-hand side of, say, (Dir) demands of the world that it contain directions, the right-hand side does not. Abstraction therefore involves a worldly asymmetry… Proponents of the asymmetric picture … insist that the right-hand side of a legitimate abstraction principle suffices for the left-hand side…
>
> How should this sufficiency statement be understood? It cannot straightforwardly be understood in terms of demands on the world. For according to the asymmetric picture, the demands of ["the direction of l_1 = the direction of l_2"] exceed those of ["l_1 is parallel to l_2"]. Of course, a proponent of thin objects will deny that the demands of the former statement substantially exceed those of the latter—but this is itself in need of explanation. I find it useful to understand the desired notion of sufficiency as a species of metaphysical grounding. (Linnebo, 2018, p. 18)

Linnebo's goal is to not merely say "ground", but rather to say something about *how* the grounding works.

3. The triangle



Identity criteria → reference: start with an appropriate equivalence relation, $\sim$ over parcels of matter; then you can introduce terms for material bodies containing parcels of matter, using this abstraction principle:

$$b(u) = b(v) \text{ iff } u \sim v$$

You could train a robot to use such terms.

Reference → objecthood:

> I claim that it suffices for our robot to refer to a body that the robot is appropriately related to some parcel of matter and that it treats two such parcels as specifications of the same body just in case they are related by the appropriate partial equivalence relation. There is no more direct way for it to "get at" a physical body. The most direct form of reference to bodies

> is constituted on the basis of a specification and a partial equivalence relation that provides a criterion of identity. (Linnebo, 2018, p. 29)

"Directness" of reference isn't really the issue; the issue is metaphysical:

> Suppose we have assigned truth-conditions to all identity statements and predications that involve some new singular terms we are trying to introduce.... *Does this attempted application of our conceptual apparatus succeed in latching on to objects?* Two different answers flow from two rival conceptions of ontology...
>
> *The rigid conception* holds that reality is "carved up" into objects in a unique way that is independent of the concepts that we bring to bear. This conception introduces an element of risk into our proposed application of our conceptual apparatus for identification and predication. Although the attempted application is logically in good order, reality may fail to cooperate. Reality may simply not contain the sorts of objects we are trying to "carve out".
>
> *The flexible conception*, on the other hand, insists that reality is articulated into objects only through the concepts that we bring to bear. And we often have some choice in this matter...
>
> ...there is no unique, privileged set of concepts in terms of which to "carve up" reality, namely the concepts that match some rigid concept-independent articulation of reality into objects. This means there is no risk that reality fails to contain objects answering to some coherent application of our apparatus for identification and predication. This coherent application "carves out" the appropriate objects. (Linnebo, 2018, p. 31)

1. What is the flexible view?

One version: "bare" quantification ($\exists$) is incoherent; only sortal-relative quantification is coherent ($\exists_F$). What are the admissible Fs?

Another version: quantifier variance.

What is the explanation of how $\Rightarrow$ works?

It isn't (fully) clear, since the flexible conception doesn't say what it is about the world that makes ontological claims nonrisky; it just says *that* they are.

What does "no risk" mean?

At most we have no-risk beyond the falsity of the metaphysics of thin objects.

4. Indefinite extensibility

Objecthood → criteria of identity:

The procedure of introducing new objects via abstraction can iterate. This is the key to set theory: starting with any objects, you can introduce an abstraction principle for sets:

For any xx and yy, $\{xx\} = \{yy\}$ iff for all z, $z \prec xx$ iff $z \prec yy$

The starting objects (xx and yy) can themselves be sets. So for for any collection of sets, there exists a set of the collection. No paradox because the resulting set isn't in the range of the original quantifiers ("for all xx")

(This is his solution to the bad company objection to neoFregeanism.)

References

Hale, Bob and Crispin Wright (2009). "The Metaontology of Abstraction." In David J. Chalmers, David Manley and Ryan Wasserman (eds.), *Metametaphysics*, 178–212. Oxford: Oxford University Press.

Linnebo, Øystein (2018). *Thin Objects: An Abstractionist Account*. Oxford: Oxford University Press.

Thomasson, Amie L. (2015). *Ontology Made Easy*. New York: Oxford University Press.