Classical Conventionalism

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Philosophy of Mathematics

"$2+2=4$ simply because of what we have decided to mean by '2, '4', and '+'"

Attractive to empiricists

Distinct from: logicism, formalism, "existence is thin"

1. Definitions transform rather than create truths (Quine)

Example of a definition:

Let 'even number' be synonymous with 'natural number divisible by 2'

Digression:

'Natural number divisible by 2' is a *complex predicate*. In formal languages, complex predicates are constructed using lambda abstraction. Then the definition can be:

Let 'E' be synonymous with '$\lambda x.Nx \,\&\, \exists y\; x = y \times 2$'

This insures that any formula with 'E' is synonymous to some formula that doesn't contain 'E'.

Without lambda abstraction, we must phrase the definition thus:

For any term, α, let $\ulcorner E\alpha \urcorner$ be synonymous with $\ulcorner \exists y\; \alpha = y \times 2 \urcorner$.

This also insures that any formula with 'E' is synonymous to some formula that doesn't contain 'E', *if* the only way 'E' can occur is as part of a formula of the form $\ulcorner E\alpha \urcorner$.

So, are logical and mathematical truths definitions?

No: definitions are speech acts, not assertions.

Ok Mr. Pickypants, maybe logical and mathematical truths are *guaranteed* by definitions.

But what does that mean?

Definitions guarantee that these pairs of sentences are synonymous:

48 is even

$\exists y\ 48 = y \times 2$

48 is even if and only if it is divisible by 2

$\exists y\ 48 = y \times 2 \leftrightarrow \exists y\ 48 = y \times 2$

But in each pair, the second sentence presumably "already" needs to be true.

2. Are mathematical truths Quine-analytic?

"Quine-analytic" sentences are those that transform into logical truths by substituting definiens for definiendum. Are mathematical truths Quine-analytic?

Argument one: '$0 \neq 1$' isn't Quine-analytic, because no matter how you define '0' and '1', the result after substituting will have the form $\alpha \neq \beta$, and no sentence of that form is a (first-order) logical truth.

Note: this assumes the definitions would be of '0' and '1' individually.

Argument two: there can't be "mechanical" definitions that map all and only truths of arithmetic to first-order logical truths, since the first-order logical truths are mechanically listable, whereas the truths of arithmetic are not.

Note: this assumes that "logical truth" in the definition of Quine-analyticity means first-order logical truth.

3. Ayer on analyticity

> I think that we can preserve the logical import of Kant's distinction between analytic and synthetic propositions, while avoiding the confusions which mar his actual account of it, if we say that a proposition is analytic when its validity depends solely on the definitions of the symbols it contains, and synthetic when its validity is determined by the facts of experience… the proposition 'Either some ants are parasitic or none are' is an analytic proposition. For one need not resort to observation to discover that there either are or are not ants which are parasitic. If one knows what is the function of the words 'either', 'or', and 'not', then one

> can see that any proposition of the form ‘Either p is true or p is not true’ is valid, independently of experience.…
>
> …the proposition “Either some ants are parasitic or none are” provides no information whatsoever about the behaviour of ants, or, indeed, about any matter of fact. And this applies to all analytic propositions. They none of them provide any information about any matter of fact. In other words, they are entirely devoid of factual content. And it is for this reason that no experience can confute them. (Ayer, 1936, pp. 78-9)
>
> …the propositions of mathematics are analytic propositions. They will form a special class of analytic propositions, containing special terms, but they will be none the less analytic for that. For the criterion of an analytic proposition is that its validity should follow simply from the definition of the terms contained in it, and this condition is fulfilled by the propositions of pure mathematics. (Ayer, 1936, pp. 81–2)

4. Ayer: epistemic analyticity

The claim in the first paragraph that anyone who understands ‘or’ and ‘not’ “can *see* …independently of experience” (my emphasis) that instances of LEM are true suggests that ‘analytic’ merely means a priority. (Compare Boghossian’s notion of (1996) “epistemic analyticity”.)

But then a sentence’s being analytic can’t explain how we know it.

5. Ayer: “no information”

At the end of the first quotation, Ayer suggests that we can know logical truths a priori because they don’t “provide any information about any matter of fact”.

But what does that mean? ‘Either some ants are parasitic or none are’ provides the information that either some ants are parasitic or none are. Here are some potential senses in which this information isn’t “a matter of fact”:

> It’s obvious
>
> It’s a logical truth
>
> It’s necessarily true

It isn’t clear that any of these statuses explains knowledge.

6. Ayer: "follows from the definition"

At the end of the second quotation, Ayer says that the truth of an analytic sentence "follows from" definitions. What does this mean?

If the "definitions" are the axioms of some branch of mathematics and "follows from" means "logically follows from", no account of our knowledge of the definitions, or of logical knowledge, has been given.

7. Ayer: "record our determination"

> . . . the reason why [analytic statements] cannot be confuted in experience is that they do not make any assertion about the empirical world. They simply record our determination to use words in a certain fashion. (Ayer, 1936, p. 84)

Does the second sentence mean that asserting:

(A) Either some ants are parasitic or none are

is really just asserting:

> I am determined to continue to use the sentence 'either some ants are parasitic or none are' in certain ways.

? That would be really weird. E.g., it would make the assertion only contingently true.

Does it instead mean that an "assertion" of (A) isn't really an assertion at all, but just an "expression" of how I am determined to use words? But then how could (A) be used as a premise in reasoning?:

(A) Either some ants are parasitic or none are

If some ants are parasitic then a certain conclusion C follows

If no ants are parasitic then that same conclusion C follows

So either way, conclusion C follows

8. Ayer: "depends solely on", and truth by convention

A final thing Ayer says is that an analytic sentence "depends solely on the definitions of the symbols it contains".

Perhaps this means that the very act of stipulating what words are to mean somehow *produces* certain truths containing those words.

This is sometimes called "truth by convention". (Compare Boghossian on "metaphysical analyticity".)

Some objections to this idea (compare Boghossian (1996)):

> You can't bring something about by stipulation, unless that something is *about* stipulation. But that some ants are parasitic or none are isn't about stipulation (it's about ants).
>
> It was the case before there were any humans around to make stipulations that either some ants are parasitic or none are.
>
> The truth of a sentence is due to two factors: for some P, i) the sentence means that P, and ii) P. Convention only plays a role in securing i), and no role in securing ii) (unless P is about conventions).

9. Quine's (1936) Lewis Carroll argument

Suppose (for the sake of argument) that pronouncing 'let it be the case that sentence S is true' produces the truth of S. But there are infinitely many logical truths, and we can't make infinitely many pronouncements.

We might try these pronouncements:

1. Every sentence of this form is to be true: if A, then if B then A
2. Every sentence of this form is to be true: If (If A, then if B then C), then (if A then B, then if A then C)
3. Every sentence of this form is to be true: If not-A then not-B, then if B then A
4. For any sentences A and B, if A is to be true, then if "if A then B" is to be true, B is to be true.

The first three correspond to three axioms for propositional logic given by Jan Łukaswiewicz; the fourth corresponds to the rule modus ponens. This axiomatization is complete.

Quine points out that you need to *use* logic to infer claims about which sentences are to be true from these stipulations. For example:

> 'If snow is white, then if grass is green then snow is white' is to be true (from 1)
>
> 'If (If snow is white, then if grass is green then snow is white), then (if snow is white then grass is green, then if snow is white then snow is white)' is to be true (from 2)
>
> If 'If snow is white, then if grass is green then snow is white' is to be true, then if 'If (If snow is white, then if grass is green then snow is white), then (if snow is white then grass is green, then if snow is white then snow is white)' is to be true, 'if snow is white then grass is green, then if snow is white then snow is white' is to be true (from 4)
>
> Therefore, 'if snow is white then grass is green, then if snow is white then snow is white' is to be true

To reach the fourth step, you need to use modus ponens and earlier steps twice. Thus "you already need modus ponens, in order to apply these conventions".

It's a little unclear exactly what Quine's argument establishes. Two possibilities:

Semantic argument You can't use 1–4 to teach logical words to someone who doesn't know any logical words. For 4 uses 'for any' and 'if…then'—such a person won't understand 4.

Epistemic argument Someone who has no logical knowledge (including lacking the ability to extend their knowledge by using logical rules of inference) can't use 1–4 to gain knowledge of logical truths, since they would need to be able to use the logical rule of modus ponens in order to do so.

One also wonders about a metaphysical argument that 1–4 can't *make it the case that* logical truths are true since modus ponens would "already need to be in effect". Unclear exactly what this argument is. I also wonder whether a conventionalist might reply:

> There's nothing magical about the words 'let it be the case that'. The idea

of truth by convention is that certain intentions to use words in a certain way make it the case that certain sentences are true. But why couldn't a speaker also have a *conditional intention*, to the effect that whenever they are treating ϕ and also $\phi \rightarrow \psi$ in a certain way, they will also treat ψ in that way? In that case these conditional intentions could make it the case that *a rule holds*.

10. Tonk

Prior's target is the view that logical inferences are "analytically valid", in that:

i) Their correctness "arises solely from the meanings of certain expressions occurring in them" (Prior, 1960, p. 38)

ii) The meanings of those expressions is constituted by the fact that we have chosen those inferences to be meaning-constituting.

The following quotation, concerning the logical word 'and', clarifies ii):

> A doubt might be raised as to whether it is really the case that, for any pair of statements P and Q, there is always a statement R [*viz.*, the conjunction of P and Q] such that given P and given Q we can infer R, and given R we can infer P and can also infer Q. But on the view we are considering such a doubt is quite misplaced, once we have introduced a word, say the word 'and ', precisely in order to form a statement R with these properties from any pair of statements P and Q. The doubt reflects the old superstitious view that an expression must have some independently determined meaning before we can discover whether inferences involving it are valid or invalid. With analytically valid inferences this just isn't so. (Prior, 1960, p. 38)

Prior says: suppose we introduce a new logical connective, TONK, governed by these inference rules:

$$\frac{A}{A \text{ TONK } B} \quad \text{(Tonk-intro)} \qquad\qquad \frac{A \text{ TONK } B}{B} \quad \text{(Tonk-elim)}$$

The view seems to imply that we have successfully introduced a new connective, 'TONK', which obeys the two rules. But then the sky is purple:

1. Snow is white (premise)
2. Snow is white TONK the sky is purple (1, TONK-intro)
3. The sky is purple (2, TONK-elim)

References

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