Structuralism

Ted Sider Philosophy of Mathematics

Structuralist slogan: "Only structure matters"

1. Set theoretic platonism and structuralism

Set theoretic platonism partially fits this slogan, since sets deserve to be defined as mathematical objects only because of their structural features.

(Thus multiple set-theoretic definitions would be adequate. E.g. we could reverse \emptyset and $\{\emptyset\}$ in the sequence of natural numbers.)

(Thus perhaps ordinary talk of natural numbers is massively ambiguous.)

But sets themselves don't fit the slogan. We are talking about particular entities when we say that set A is a member of set B.

2. Ante rem structuralism

To more closely adhere to the slogan, ante rem structuralists posit new entities, *structures* and *positions in structures*.

(A baseball defense is a kind of structure, containing the positions of shortstop, center field, etc. This structure and positions exist independently of being exemplified by particular baseball players.)

Ante rem structuralists say that mathematical objects *are* positions in structures. E.g., the number 0 is the first position in the *natural number structure*—the structure that is exemplified by any sequence of entities with the same structure as the natural numbers.

Objection: why is the first sequence of positions a better candidate to be the natural numbers than the second or the third?:

first position	second position	third position	•••
Julius Caesar	first position	second position	•••
second position	first position	third position	•••

In reply, ante rem structuralists say things like: "There is nothing more to 'positions' than how they occur in their defining structures; they have no existence or identity outside of those structures".

3. Structuralism I: positions have no properties

Better: positions have no fundamental, monadic properties, and no proper parts.

Objections:

First position and second position could still swap places.

Even sets would count as "positions in structures"

4. Structuralism II: essence

"For any consistent theory, there is exactly one collection of entities that are (nonderivatively) essentially structured as the theory says"

(Close variant: "for any consistent theory, there is exactly one collection of entities and perfectly natural relations that satisfy the theory")

Objection: this theory is massively unparsimonious.

5. An unsolvable problem?

Any view about the foundations of mathematics will put forward particular entities structured in some particular way to undergird mathematics; so it seems that no such view could comply with the slogan.

Possible solution: the slogan is about methodology, not metaphysics. Mathematics only *cares* about structure, but more might be *true* beyond structure.