Philosophy of Mathematics

Ted Sider, Spring 2025
Tuesdays 9-12

This graduate seminar will be an introduction to philosophy of mathematics, aimed at people who have never studied the subject. No special background in either logic or mathematics will be assumed.

At least half of the course will be a survey of traditional questions about the epistemology and metaphysics of mathematics (e.g., how can we have knowledge of mathematical entities if they are abstract and thus causally isolated from us?) The survey will begin with a brief look at some older views (Plato, Kant, Mill), then turn to the main views about the foundations of mathematics that emerged at the end of the nineteenth century (Frege's logicism, formalism, and intuitionism), and finally discuss some more recent views (neo-logicism, set-theoretic platonism, structuralism). Interludes on the history of mathematics will be interspersed. The main text for this survey will be a draft of an introductory textbook on philosophy of mathematics that I'm writing.

After the survey, we'll discuss some live issues in the contemporary debate, focusing on the borderline between philosophy of mathematics and metaphysics.

Requirements

If taking for research credit: either two short (roughly 10–12 pages) papers, due 3/25 and 5/13, or one term paper (roughly 20 pages), due 5/13. If not taking for research credit: two short expository papers (roughly 6–8 pages), due 3/25 and 5/13.

Readings

All required readings will be distributed via Canvas. Near the beginning of the semester it might be nice to read through *Mathematics: A Very Short Introduction*, by Timothy Gowers—a wonderful (and short) book on the nature and practice of mathematics. But this is only a suggestion.

Course websites

https://tedsider.org/teaching/mathematics_25/phil_math.html

https://rutgers.instructure.com/courses/338217

Schedule (under construction)

1/21 Introduction; Plato; Kant; nonEuclidean geometry.

REQUIRED READING: Sider (2025, Chapters 1–4)

1/28 Mill; Modern mathematics and logic; Frege.

REQUIRED READING: Sider (2025, Chapters 5–7, Section 8.1)

FURTHER READING: Mill: Skorupski (1989, Chapters 3–5); Frege: George and Velleman (2002, Chapter 2)

2/4 Frege continued; the foundational crisis; formalism.

REQUIRED READING: Sider (2025, Chapter 9; Sections 10.1–10.3)

FURTHER READING: Ferreirós (2008)

2/11 Formalism continued; intuitionism; Gödel.

REQUIRED READING: Sider (2025, Sections 10.4, 10.5; Chapter 11)

FURTHER READING: Intuitionism: George and Velleman (2002, Chapter 4); Hilbert: Zach (2023); Gödel: Smith (2007)

2/18 Set-theoretic platonism; structuralism.

REQUIRED READING: Sider (2025, Chapters 12–13)

FURTHER READING: Benacerraf (1965); Resnik (1997, part 3); Shapiro (1997, part 2); Linnebo (2008); Linnebo and Pettigrew (forthcoming); Sider (2020, sections 3.6, 3.10–3.12)

2/25–3/4 Conventionalism

REQUIRED READING: Ayer (1936, chapter IV); Quine (1936); Warren (2020, Introduction, Chapter 1, 3, 4 (sections I–III), 7)

FURTHER READING: Carnap (1937, pp. ??); Quine (1960); Thomasson (2015)

3/11–3/25 Neo-logicism

REQUIRED READING: Sider (2025, Section 8.2); Rayo (2013, chapters 1–3); Linnebo (2018, Chapters 1, 2, 4)

FURTHER READING: Wright (1983); Hale and Wright (2001); Field (1984); Sider (2007); Hawley (2007); Hale (2007); Hale and Wright (2009); Eklund (2020)

4/1–4/8 Mathematical plenitude

REQUIRED READING: Balaguer (1998, pp. 5–8; Chapter 3); Hamkins (2012)

FURTHER READING: Clarke-Doane (2020); Eklund (2006)

4/15 Grounding

REQUIRED READING: Donaldson (2017)

FURTHER READING: Rosen (2011, 2016); deRosset and Linnebo (2023); Schwartzkopff (2011)

4/22–4/29 Field's nominalism.

REQUIRED READING: Field (2016, Chapters 1–5)

FURTHER READING: Malament (1982); Field (1989); Shapiro (1983); Burgess and Rosen (1997, pp. ??)

Further topics

Here are some further topics that I won't cover in class, but which might be suitable for a term paper.

Set-theoretic realism and perception: Maddy (1990, chapter 2)

Higher-order logic and mathematics: Dorr (2025); Goodsell and Yli-Vakkuri (manuscript)

Absolute generality: Rayo and Uzquiano (2007); Fine (2007); Linnebo (2022)

Parallels between mathematics and metaethics: Clarke-Doane (2020)

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