KRIPKE

Ted Sider
Modality seminar

1. Kripke's semantics for modal logic

$\Box A$ "Necessarily, A"

$\Diamond A$ "Possibly, A"

LEWIS'S SYSTEM K:

- *Rules of inference*: modus ponens, plus the rule of *necessitation*, which lets you infer $\Box A$ from A.
- *Axioms*: those of propositional logic, plus:

$$\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B) \tag{K}$$

LEWIS'S SYSTEM S4:

- *Rules of inference*: same as K
- *Axioms*: those of K, plus:

$$\Box A \rightarrow A \tag{T}$$
$$\Box A \rightarrow \Box\Box A \tag{S4}$$

Proof-theoretic approach A logically implies B iff you can reason, using specified axioms and rules of inference, from A to B

Model-theoretic approach A logically implies B iff B is true in every model in which A is true

propositional logic model: an assignment of truth-values to sentence letters.

predicate logic model: a domain and an interpretation function that assigns denotations to names and extensions to predicates.

modal logic model: a set of "possible worlds", each one containing an entire model of the old sort.

$\Box A$ is true at world w iff A is true at every world accessible from w

$\Diamond A$ is true at world w iff A is true at some world accessible from w

Promising features of Kripke semantics:

- Explaining logical features of modal logic (duality of $\Box$ and $\Diamond$; logical truth of axiom K)

- Correspondence of Lewis's systems to formal features of accessibility

A cautionary note: what good is all this if $\Box$ doesn't *mean* truth in all worlds?

2. Kripke's *Naming and Necessity*

Early defenders of modal logic, for example C. I. Lewis, thought of the $\Box$ as meaning analyticity. Quine argued that de re modal claims, thus understood, made no sense. $\exists x \Box F x$, thus understood, means that for some x, 'Fx' is analytic. But analyticity is a matter of meaning, and 'Fx' has no meaning, even given an assignment of an object to x. The object could be denoted by many terms α, and $F\alpha$ would be analytic for some and not for others. Quine concluded:

> The only hope [of making sense of quantifying into modal contexts] lies in … insisting that the object x in question is necessarily greater than 7. This means adopting an invidious attitude toward certain ways of uniquely specifying x …and favoring other ways as somehow better revealing the "essence" of the object…

> Evidently this reversion to Aristotelian essentialism…is required if quantification into modal contexts is to be insisted on….

> Essentialism is abruptly at variance with the idea, favored by Carnap, Lewis, and others, of explaining necessity by analyticity. For the appeal to analyticity can pretend to distinguish essential and accidental traits of an object only relative to how the object is specified, not absolutely. Yet the champion of quantified modal logic must settle for essentialism…

> The upshot of these reflections is meant to be that the way to do quantified modal logic, if at all, is to accept Aristotelian essentialism. To defend Aristotelian essentialism, however, is not part of my plan. Such a philosophy is as unreasonable by my lights as it is by Carnap's or Lewis's. And in conclusion I say, as Carnap and Lewis have not: so much the worse for

quantified modal logic. By implication, so much the worse for unquan-
tified modal logic as well; for, if we do not propose to quantify across
the necessity operator, the use of that operator ceases to have any clear
advantage over merely quoting a sentence and saying that it is analytic.
(Quine, 1953, 155–6)

Kripke's defense of quantified modal logic was basically to embrace the "Aris-
totelian Essentialism" that Quine assumed was beyond the pale:

> Some philosophers have distinguished between essentialism, the belief in
> modality de re, and a mere advocacy of necessity, the belief in modality
> de dicto. Now, some people say: Let's give you the concept of necessity.
> A much worse thing, something creating great additional problems, is
> whether we can say of any particular that it has necessary or contingent
> properties…only a statement or a state of affairs…can be either necessary
> or contingent. Whether a particular necessarily or contingently has a
> certain property depends on the way it's described.…What is Quine's
> famous example? If we consider the number 9, does it have the property
> of necessary oddness? Has that number got to be odd in all possible
> worlds? Certainly it's true in all possible worlds…that nine is odd. Of
> course, 9 could also be equally well picked out as the number of planets. It
> is not necessary, not true in all possible worlds, that the number of planets
> is odd… And so it's thought: Was it necessary or contingent that Nixon
> won the election? (It might seem contingent, unless one has some view of
> some inexorable processes…) But this is a contingent property of Nixon
> only relative our referring to him as 'Nixon'…if we designate Nixon as
> 'the man who won the election in 1968', then it will be a necessary truth,
> of course, that the man who won the election in 1968, won the election in
> 1968. Similarly, whether an object has the same property in all possible
> worlds depends not just on the object itself, but on how it is described.
> So it's argued.
>
> It is even suggested in the literature, that though a notion of necessity may
> have some sort of intuition behind it (we do think some things could have
> been otherwise; other things we don't think could have been otherwise),
> this notion [of a distinction between necessary and contingent properties]
> is just a doctrine made up by some bad philosopher, who (I guess) didn't
> realize that there are several ways of referring to the same thing. I don't
> know if some philosophers have not realized this; but at any rate it is very
> far from being true that this idea [that a property can meaningfully be held
> to be essential or accidental to an object independently of its description]

is a notion which has no intuitive content, which means nothing to the ordinary man. Suppose that someone said, pointing to Nixon, 'That's the guy who might have lost'. Someone else says 'Oh no, if you describe him as "Nixon", then he might have lost; but of course, describing him as the winner, then it is not true that he might have lost'. Now which one is being the philosopher, here, the unintuitive man? It seems to me obviously to be the second. The second man has a philosophical theory. The first man would say, and with great conviction, 'Well, of course, the winner of the election might have been someone else. The actual winner, had the course of the campaign been different, might have been the loser, and someone else the winner; or there might have been no election at all. So, such terms as "the winner" and "the loser" don't designate the same objects in all possible worlds. On the other hand, the term "Nixon" is just a name of this man'. When you ask whether it is necessary or contingent that Nixon won the election, you are asking the intuitive question whether in some counterfactual situation, this man would in fact have lost the election. If someone thinks that the notion of a necessary or contingent property (forget whether there are any nontrivial necessary properties [and consider] just the meaningfulness of the notion) is a philosopher's notion with no intuitive content, he is wrong. Of course, some philosophers think that something's having intuitive content is very inconclusive evidence in favor of it. I think it is very heavy evidence in favor of anything, myself. I really don't know, in a way, what more conclusive evidence one can have about anything, ultimately speaking. But, in any event, people who think the notion of accidental property unintuitive have intuition reversed, I think. (Kripke, 1972, 39–42)

This argument, together with Kripke's arguments for the distinctness of necessity and apriority, gave us our contemporary notion of "metaphysical necessity".

References

Kripke, Saul (1972). "Naming and Necessity." In Donald Davidson and Gilbert Harman (eds.), *Semantics of Natural Language*, 253–355, 763–9. Dordrecht: D. Reidel. Revised edition published in 1980 as *Naming and Necessity* (Harvard University Press, Cambridge, MA).

Quine, W. V. O. (1953). "Reference and Modality." In *From a Logical Point of View*, 139–59. Cambridge, MA: Harvard University Press.